THE YEAR 11 MATHS EXAM 1961

I'll lay you London to a brick that if you passed this exam in 1961, unless you're a maths teacher you won't have a snowball's chance in hell of passing it in 2013. This stuff is as irrelevant today as it was in 1961. Just a sheer waste of educational time. If telling adolescents that they need to do maths to progress their life choices isn't a fraud, then it's just plain bunkum.

MATHEMATICS I (1961).

1. (i) Evaluate, using tables:

   (a) $\cos(-156°40')$,

   (b) $\cot 148°34'$,

   (c) $\dfrac{1}{\log \sin 40°}$.

   (ii) Find the values of A between 0° and 360° for which $2 \sec 2A + 5 = 0$.

   (iii) Simplify $\dfrac{\cos(180° - A) + \sin(270° - A)}{\tan(90° + A)}$ and find its value without using tables if $\tan A = -\frac{3}{4}$ and $0° < A < 180°$.

2. Draw the graph of $y = \dfrac{7x + 4}{2x^2 + 1}$ for values of x from $x = -3$ to $x = +3$ and hence determine the greatest and least values of y in this range and the corresponding values of x. If the graph were plotted beyond $x = 3$, would it cut the x axis again? Give reasons for your answer.

   Use your graph to solve the equation $\dfrac{7x + 4}{2x^2 + 1} = 1{\cdot}5$ and check your solutions by solving the equation algebraically.

3. State carefully, without proof, three sets of conditions under which two non-congruent triangles are similar.

   ABC and PQR are two similar triangles in which A and P, B and Q, C and R are corresponding vertices.

   (a) If H is a point on AB and K a point on PQ such that $\dfrac{AH}{HB} = \dfrac{PK}{KQ}$ prove that $\dfrac{HC}{KR} = \dfrac{BC}{QR}$.

   (b) If r and R are the radii of the inscribed circles of triangles ABC and PQR, prove that $\dfrac{r}{R} = \dfrac{BC}{QR}$.

4. A and B are two towns joined by a straight road and B is 5 miles due east of A. A town C is N. 40°E. from A and N. 15°20'W. from B. Another town D is N. 60°E. from A and its perpendicular distance from AB is 1·5 miles. Find, by calculation, the distances CA, AD and CD. Calculate also, to the nearest degree, the bearing of C from D.

5. (i) Find the range of values of x for which $\dfrac{x+3}{x-4} > 1{\cdot}5$.

   (ii) Find, in terms of c, the minimum value of $2x^2 + 3x + c^2$. Hence determine the values of c for which the graph of $y = 2x^2 + 3x + c^2$ lies entirely above the x axis.

   (iii) Prove that in any triangle ABC, $\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ where a, b and c are the sides opposite A, B and C respectively.

   If $\cos B = \dfrac{\sin A}{2 \sin C}$ prove that the triangle must be isosceles.

6. Prove that a straight line that cuts two sides of a triangle proportionally is parallel to the third side.

ABC is a triangle right-angled at A, and AD is perpendicular to BC, meeting it at D. The internal bisector of angle ABC meets AD at P and the internal bisector of angle DAC meets BC at Q. Prove that PQ is parallel to AC.

If $AB = 3''$ and $AC = 4''$, calculate the length of AD, the ratio $\dfrac{DQ}{QC}$ and the length of PQ.

7. (i) The side AB of a parallelogram $ABCD$ is produced to E so that $AB = BE$ and F is the middle point of BE. The line DF meets BC at X. Prove that X is the common centroid of the triangles ACE and DBE.

(ii) If $a = 10^x$, $b = 10^{2x}$ and $c = 10^{1-x}$

(a) express $\log \dfrac{10\sqrt{b}}{a^3 c}$ in terms of x,

(b) find the value of c if $b - 2a = 3$.

(iii) If x is any positive number, show that $x + x^{-1}$ is never less than 2.

If $\dfrac{1+x}{1-x} = \dfrac{\sqrt{3}}{\sqrt{2}}$, prove that $x + x^{-1} = 10$.

But wait, there’s more.

Numeracy by all means. Arithmetic, definitely – especially tables and mental. The average year 11 student these days can’t tell you what 42 divided by 7 is, fer chrissakes! Basic accounting, certainly. Cards, absolutely. If the mind still needs stimulating then try chess, sodoku, scrabble or any sporting or manipulative skill.

Betcha can’t do the Maths II paper either.

MATHEMATICS II.

1. The first two terms of a progression are 14 and 9·8.

Find the tenth term and the sum of ten terms if the progression is (a) arithmetic, (b) geometric.

Find the number of terms in the arithmetic progression that are greater than -100; and the number of terms in the geometric progression that are greater than 1.

2. (i) If $\tan A=\frac{3}{4}$, $\tan B=\dfrac{-15}{8}$, A is acute and B is obtuse, find without using tables the values of:

(a) $\cos(A-B)$ (b) $\sin(A+B)$

(ii) Prove that:

$$\tan(x+60^\circ)\,.\,\tan(x-60^\circ)=\frac{1-4\cos^2 x}{1-4\sin^2 x}.$$

(iii) Solve the equation:

$\sin(x+60^\circ)=2\cos(x+60^\circ)$

for values of x between -180° and 180°.

3. (i) Show that $(-6, -3)$, $(3, -4)$ and $(4, 5)$ may be three vertices of a square, and determine the coordinates of the fourth vertex.

(ii) $A(3,4)$, $B(-3,-2)$ and $C(5,-2)$ are the vertices of a triangle ABC. Write down the equations of the lines:

(a) through A, perpendicular to BC,

and (b) through B, perpendicular to AC,

and determine the coordinates of H, the orthocentre of the triangle ABC. Verify that CH is perpendicular to AB.

4. (i) Draw a line AB, 4·5 inches long. Using a geometric construction find a point P in AB such that $AP^2=\frac{1}{3}AB^2$. State and prove your construction.

(ii) A is any point on the circumference of a circle with centre O. On OA as diameter a second circle is drawn. At any point P, on OA, PQR is drawn perpendicular to OA meeting the smaller circle in Q and the other circle in R.

Prove that

(i) $AQ^2=AP.AO$.

(ii) $AR^2=2AQ^2$.

5. (i) Two variables x and y are connected by a relation $x=cy^n$ where c and n are constants. If $x=45$ when $y=2$ and $x=80$ when $y=1·5$, find c and n.

(ii) The following are a set of corresponding values of two variables p and q.

p	4·3	15·5	28·9	53·0
q	0·8	2·5	3·6	5·0

Show graphically that p and q are related by an equation of the form $p=aq^2+b$.

Find, by readings from the graph, the values of a and b.

6. (i) Without using tables:

(a) express $\cos(x+25°)-\cos(x+85°)$ as the sine of an angle.

(b) evaluate $\sin 105° \cos 15°$.

(c) prove that $\cos 25°+\cos 95°+\cos 145°=0$.

(ii) Solve the equation:

$3 \sin 2x=\cos 2x+1$

for values of x between $-180°$ and $180°$.

7. (i) Prove that the three perpendiculars drawn from the vertices to the opposite sides of a triangle are concurrent.

(ii) ABC is a triangle, right-angled at B. X and Y are points on AB and AC respectively so that $AX.AB=AY.AC$.

If CB and YX produced meet in D, and CX produced meets AD in Z, prove that:

(a) CZ is perpendicular to AD

(b) $DZ.DA=DB.DC$

and (c) $DZ.DA+CY.CA=CD^2$.

There, I told you it was bunkum.

Think what you could have been doing if you hadn't been cooped up in a cage for 8 hours a week doing maths at school and another 6 hours a week doing it at home.